Overview

The AWF (analytical wall function) originally proposed by Craft et al. (2002) is slightly modified and applied to computations of a 3D diffuser flow (Case 13.2, Diffuser 1: Cherry et al., 2008) by the nonlinear eddy viscosity model (Craft, Launder and Suga, 1996) and the TCL second moment closure (Craft and Launder, 2001). Although the original form of the AWF does not contaminate flow field results, it sometimes leads to unphysical heat transfer distribution near a corner of a 3D duct flow, particularly when it is coupled with a second moment closure. The present study thus modifies the AWF form and evaluates its performance in a 3D square sectioned U-duct flow as well as the test case flow. The results of Case 13.2, Diffuser 1 clearly indicate that the TCL model with the present AWF performs reasonably well though the nonlinear eddy viscosity model only slightly improves poor results of the standard \( k-\varepsilon \) model.

Analytical Wall-Function

In the AWF, the wall shear stress and scalar flux are obtained through the analytical solution of simplified near-wall versions of the transport equations for the wall-parallel momentum and scalar. The main assumption required for the analytical integration of the transport equations is a modelled variation of the turbulent viscosity \( \mu_t \) over a wall-adjacent computational-cell. For smooth wall flows, this is done using \( y' \) as the thickness of the viscous sub-layer, and assuming that \( \mu_t \) is zero for \( y < y' \) and then increases linearly:

\[
\mu = \alpha \mu (y'/y'_{max})^2, \quad \text{where} \quad y'_{max} = \frac{y k_p^{1/2}}{\nu}, \quad \alpha = c_i c_{\mu}, \quad c_i = 2.55 \quad \text{and} \quad c_{\mu} = 0.09, \quad \text{and} \quad k_p, \nu, y, \text{and} \ k_p \text{ are respectively the molecular viscosity, the kinematic viscosity, the wall normal distance and the turbulence energy at the node } P.
\]

Then, with the assumption that the right hand side terms can be constant over the cell, the simplified momentum and scalar equations in the wall adjacent cell:

\[
\frac{\partial}{\partial y'} \left( \mu + \mu_t \right) \frac{\partial U}{\partial y'} = \frac{\nu}{k_p} \frac{\partial}{\partial x} \left( \rho U U \right), \quad (1)
\]

\[
\frac{\partial}{\partial y'} \left( \frac{\mu}{Pr} + \Gamma \right) \frac{\partial \Theta}{\partial y'} = \frac{\nu}{k_p} \frac{\partial}{\partial x} \left( \rho U \Theta \right), \quad (2)
\]

can be easily integrated analytically to form the boundary conditions of the momentum and the scalar at the wall, namely the wall shear stress and scalar flux. Note that the coordinate directions \( x, y \) correspond to the streamwise (wall parallel) and wall-normal directions, respectively. (See the original paper or Suga et al. (2006) for the detailed treatments.)

Since the original forms of the AWF are obtained in 2D wall parallel flows, it is reasonable that 3D applications of such a model require further discussions. When the 3D square sectioned U-bend duct flow (Fig.1) is considered, the secondary flows near the duct corners are typically important and produce very different conditions from those in the 2D wall parallel flows. In such a case, the streamwise direction differs from the wall parallel direction as shown in Fig.2 and the gradient in the streamwise direction is:

\[
\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial \phi}{\partial \zeta} \frac{\partial \zeta}{\partial x}, \quad (3)
\]

where \( x \) is the streamwise direction which is treated as the wall parallel direction in the original AWF. Due to the large velocity gradient in the \( \zeta \) direction, the convection terms in the equations (1) and (2) have peaky distribution near the corners as shown in Fig.3. Thus, the predicted Nusselt number distribution has unphysical peaks as in Fig.4. In order to eliminate those kinked profiles, the present study introduces a damping function \( f_3 \) to the right hand side terms of equations (1) and (2) as:
The form of \( f_{S} \) is
\[
f_{S} = \exp(-\hat{S}/A_{S}), \quad \hat{S} = \frac{k}{\varepsilon} \sqrt{\frac{1}{2} (S_{y} h_{t} h_{t})^{2}},
\]
where \( S_{y} = \partial U_{j}/\partial x_{i} + \partial U_{j}/\partial x_{i} \) and the subscripts \( i, j \) follow the wall coordinate. The vector \( h_{t} \) is defined as \( h_{t} = (1, 0, 1) \). The presently used coefficient is \( A_{S} = 0.5 \). This slight modification removes the unphysical profiles in the Nusselt number distribution as shown in Fig.4.

### Computation Methods and Results

The presently used turbulence models for the core flow regions are the standard \( k-\varepsilon \) model, the cubic nonlinear \( k-\varepsilon \) model of Craft, Launder and Suga (1996) (CLS model) and the TCL second moment closure of Craft and Launder (2001). The CLS model consists of the following model equation:
\[
\mu + \mu = \rho + \frac{\partial}{\partial x} \left( \mu + \frac{\partial \Theta}{\partial x} \right)
\]
which is the cubic stress-strain relation and \( \Omega_{ij} = \partial U_{ij}/\partial x_{i} \). The TCL model consists of the cubic pressure-strain relation of
\[
\phi_{ij1} = -c_{1} \varepsilon \left[ a_{ij} + c_{1}' \left( a_{ij} a_{jk} - \frac{1}{3} A_{S} \delta_{ij} \right) \right] - \sqrt{A} e_{ij}
\]
where \( A_{S} = C_{2} \delta_{ij} \cdot k - \nu_{t} \cdot S_{ij} + \text{Hot}_{ij} \)
\[
\text{Hot}_{ij} = c_{2} \varepsilon \left[ (S_{ij} \Omega_{ij} - \frac{1}{2} S_{ij} \Omega_{ij} \Omega_{ij}) + c_{2} \nu \tau (\Omega_{ij} \Omega_{ij} + \Omega_{ij} \Omega_{ij}) + c_{3} \nu \tau (\Omega_{ij} \Omega_{ij} \Omega_{ij} - \frac{1}{2} S_{ij} \Omega_{ij} \Omega_{ij}) \right]
\]
which is the cubic stress-strain relation and \( \Omega_{ij} = \partial U_{ij}/\partial x_{i} - \partial U_{ij}/\partial x_{i} \). The TCL model consists of the cubic pressure-strain relation of
\[
\phi_{ij2} = -0.6 \left( P_{ij} - \frac{1}{3} P_{kk} \delta_{ij} \right) + 0.3 a_{ij} P_{kk} - 0.2 \left[ \frac{u_{ji} u_{ij}}{k} S_{ij} - \frac{u_{i} u_{i}}{k} \left( \frac{\partial U_{ij}}{\partial x_{i}} + \frac{\partial U_{ij}}{\partial x_{i}} \right) \right]
\]
where \( A_{ij} = C_{2} \delta_{ij} \cdot k - \nu_{t} \cdot S_{ij} + \text{Hot}_{ij} \)
\[
\Phi_{ij} = c_{1}' \left[ \frac{7}{15} - \frac{A_{ij}}{4} \right] \left( P_{ij} - \frac{1}{3} P_{kk} \delta_{ij} \right) + \frac{1}{2} \left( \frac{a_{ij} a_{ij}}{k} \right) \Phi_{kk}
\]
and
\[
\Phi_{ij} = -0.5 a_{ij} a_{kk} P_{kk} + 0.1 \left[ \frac{u_{iij} u_{ij}}{k} S_{ij} - \frac{u_{i} u_{i}}{k} \left( \frac{\partial U_{ij}}{\partial x_{i}} + \frac{\partial U_{ij}}{\partial x_{i}} \right) \right]
\]
where \( A_{ij} = C_{2} \delta_{ij} \cdot k - \nu_{t} \cdot S_{ij} + \text{Hot}_{ij} \)

The presently used computation code is the STREAM (Lien and Leschziner, 1994) and the 3rd order MUSCL type upwind scheme is used for convection terms. Fig.5 shows the computational grid used for the computations of Case 13.2 Diffuser 1. Since the present computations use the AWF, a relatively coarse grid consisting of non-uniform \( 251 \times 21 \times 41 \) node points is applied. The inlet flow condition is obtained by solving a fully developed rectangular duct flow.

Fig.6 compares the streamwise mean velocity profiles in the several sections of the three spanwise plane.
sections. Although the CLS model tends to improve the results of the standard $k-\varepsilon$ model, the agreement with the experimental data is still very poor. The predicted profiles by the TCL model generally well accord with the experimental data while those at some sections still have large margins to be improved (e.g. at $x/H=12,15$ of $z/B=3/4$).

**Concluding Remarks**

1) The present modification for the analytical wall-function can improve unphysically predicted heat transfer profiles near a corner of a 3D duct flow by the original form.

2) The predictive performance of the TCL model with the present AWF is generally satisfactory in the turbulent 3D diffuser flow which is difficult to predict reasonably by the eddy viscosity models presently applied.

**References**


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**Fig. 1** Square sectioned U-bend duct.

**Fig. 2** Velocity vector near a corner of a 3D duct flow.

**Fig. 3** Distribution of the convection terms of the near wall cells.
Fig. 4 Nusselt number distribution of the U-bend duct.

Fig. 5 Computational grid for Case 13.2 Diffuser 1.
Fig. 6 Streamwise mean velocity distribution of Diffuser 1.